1503/203
MATHEMATICS II
Oct./Nov. 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN AUTOMOTIVE ENGINEERING

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Mathematical table;*
- *Non-programmable scientific calculator;*
- *Answer booklet.*

*Answer any **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
ALL questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

7. (a) Solve the equation $8x^2 + 9x + 1 = 0$, by factorisation.

(5 marks)

(b) By completing the square, determine the constants p, q and r such that

$4x^2 + 7x + 6 = p(x + q)^2 + r$.

(7 marks)

(c) Solve the equation

$$x + 5 - \frac{1}{x+4} = 1.$$

(8 marks)

8. (a) Given that $u = 1 - 2xy - x^2 + y^2$, show that $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$.

(5 marks)

(b) Find the stationary points of the function $Z = 9 - 2x + 4y - x^2 + y^2$, and determine their nature.

(8 marks)

(c) Solve the equation:

$2\cosh x + \sinh x = 2$, correct to four decimal places.

(7 marks)

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4. (a) Given that $y = \frac{1}{x+2}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Find the first derivative of

$f(x) = \cos\left(\frac{x+1}{2-x}\right)$.

(5 marks)

(c) Find the stationary points of the function

$f(x) = x^3 + 6x^2 + 9x - 8$, and determine their nature. (10 marks)

5. (a) Evaluate the integrals:

(i) $\int_0^{\frac{\pi}{2}} (3\cos x + \sin^2 x)\, dx$. (3 marks)

(ii) $\int_0^1 \frac{x + x^2 + x^{\frac{1}{2}}}{\sqrt{x}}\, dx$

(5 marks)

(b) Show that $\int \cot^2\theta d\theta = c - \theta - \cot\theta$, where c is an arbitrary constant of integration. (4 marks)

(c) Sketch the region bounded by the straight lines x = 0, x = y and y = 1, and use integration to find the area of the region. (8 marks)

6. (a) Find the angle between the vectors

$\underline{A} = 3\underline{i} - 2\underline{j} + 5\underline{k}$ and $\underline{B} = 4\underline{i} - 7\underline{j} + 8\underline{k}$

(4 marks)

(b) Determine the value of the constant **a** if the three vectors

$\underline{A} = 2\underline{i} - \underline{j} + a\underline{k}$, $\underline{B} = -2\underline{i} - 2\underline{j}$ and $\underline{C} = 4\underline{i} + 2\underline{j} - 3\underline{k}$ are coplanar.

(5 marks)

(c) Temperature distribution in a factory is given by the scalar function.

$T = 2x^2y + 3xz$

Determine, at the point (1, 3, -2),

(i) grad T

(ii) curl grad T

(iii) div grad T (11 marks)

1. (a) Given the matrices $A=\begin{bmatrix}-1 & 2\\ 2 & -1\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2 & 3\\ 6 & -1 & 2\end{bmatrix}$,

determine $(AB)^T$

(4 marks)

(b) Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 4\\ 4 & 1 & 3\\ 2 & 1 & 1\end{bmatrix}$. (6 marks)

(c) Three forces F_1, F_2, and F_3 acting at a point in a mechanical system satisfy the simultaneous equations

$F_1 + F_2 + F_3 = 9$
$F_1 - 2F_2 + 2F_3 = 7$
$3F_1 + F_2 + F_3 = 15$

Use Cramer's rule to determine the values of the forces. (10 marks)

2. (a) Write down the sixth term in the binomial expansion of $(3x + 2y)^{10}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{1}{3}$. (5 marks)

(b) Use the binomial theorem to find the value of $\frac{1}{\sqrt{1.01}}$, correct to six decimal places. (4 marks)

(c) (i) Show that, for very small values of x,

$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$.

(ii) By setting $x=\frac{1}{26}$ in question 2(c)(i) above,

determine the value of $\sqrt{27}$, correct to four decimal places.

(11 marks)

3. (a) (i) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(ii) By setting $\theta=\frac{\pi}{3}$ in (i) above, determine the value of $\sin\left(\frac{\pi}{3}\right)$ without using a calculator or tables. Leave surds in the answer. (9 marks)

(b) Prove the identities:

(i) $\frac{\cos B - \cos C}{\sin B - \sin C} = -\tan\left(\frac{B+C}{2}\right)$.

(ii) $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\,\text{cosec}\,\theta$

(6 marks)

(c) Solve the equation:

$\sin x + \sin 3x + \sin 5x = 0$, for values of x between 0° and 180° inclusive. (5 marks)